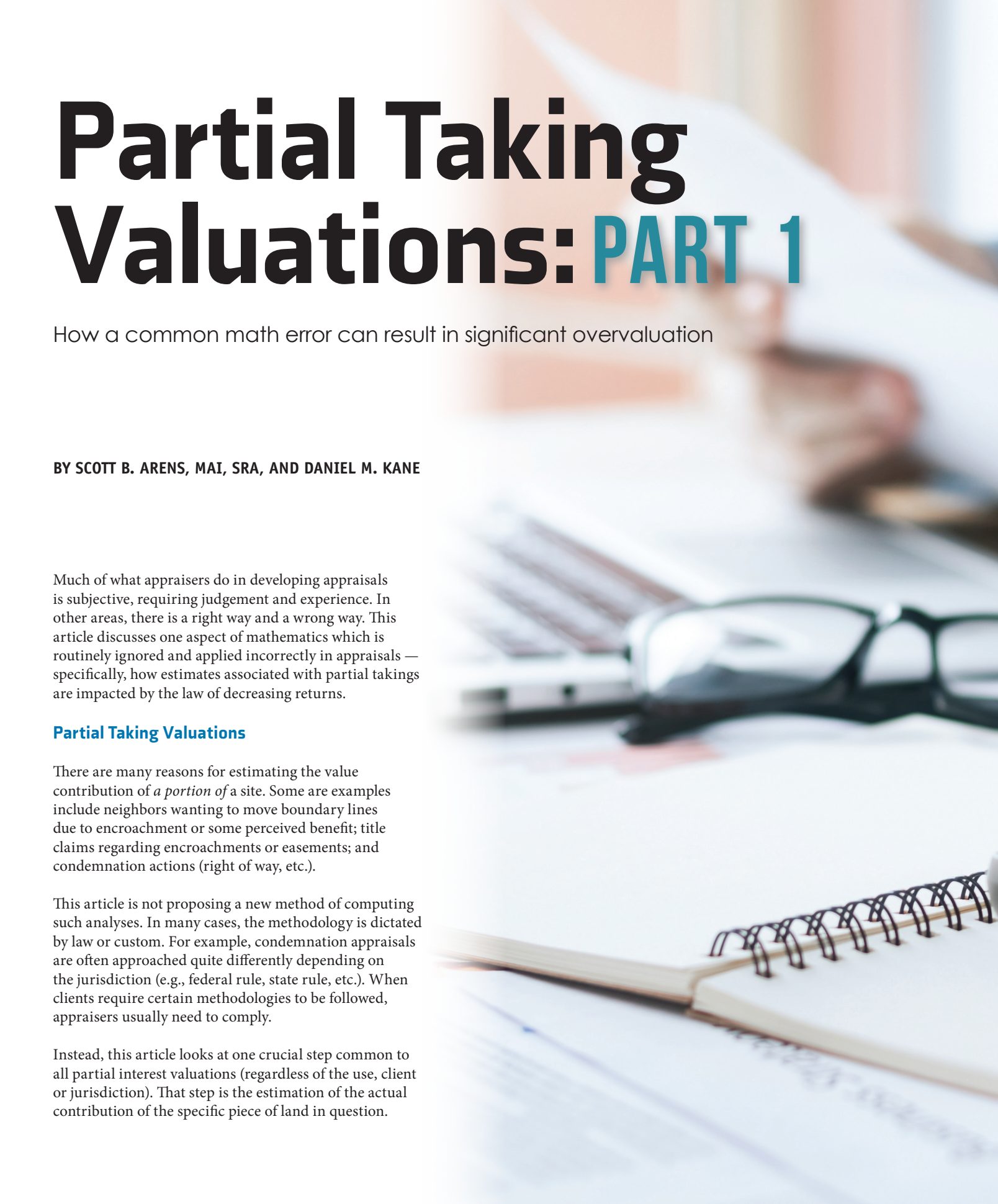
Partial Taking Valuations: PART 1

How a common math error can result in significant overvaluation



BY SCOTT B. ARENS, MAI, SRA, AND DANIEL M. KANE

Much of what appraisers do in developing appraisals is subjective, requiring judgement and experience. In other areas, there is a right way and a wrong way. This article discusses one aspect of mathematics which is routinely ignored and applied incorrectly in appraisals — specifically, how estimates associated with partial takings are impacted by the law of decreasing returns.

Partial Taking Valuations

There are many reasons for estimating the value contribution of *a portion of* a site. Some are examples include neighbors wanting to move boundary lines due to encroachment or some perceived benefit; title claims regarding encroachments or easements; and condemnation actions (right of way, etc.).

This article is not proposing a new method of computing such analyses. In many cases, the methodology is dictated by law or custom. For example, condemnation appraisals are often approached quite differently depending on the jurisdiction (e.g., federal rule, state rule, etc.). When clients require certain methodologies to be followed, appraisers usually need to comply.

Instead, this article looks at one crucial step common to all partial interest valuations (regardless of the use, client or jurisdiction). That step is the estimation of the actual contribution of the specific piece of land in question.

Decreasing Returns

Decreasing returns is defined in "The Dictionary of Real Estate Appraisal: 7th Edition" as follows:

> *The concept that successive increments of one or more agents of production added to fixed amounts of the other agents will enhance income or value (in dollars, benefits, or amenities) at an increasing rate until a maximum return is reached. Beyond a certain point, each additional unit will add less income or value than the unit before it. Also called law of increasing returns or law of decreasing returns.*

When doing partial interest valuations, the presence of decreasing returns profoundly changes the required methodology. Failing to understand and address these changes can result in conclusions that are not just wrong but often terribly wrong.

Partial Taking Case Study: Commonly Used Valuation Method

The following hypothetical example uses the common method of arriving at the estimated contribution to value of a strip of land. As will be shown, the approach results in mathematically flawed conclusions. Since the market data has been designed to be otherwise "perfect," the flaw can be seen more clearly. Under normal circumstances, the imperfection of market data tends to obfuscate the problem.

The Assignment

The subject property is a 10,000-square-foot vacant residential site in a highly conforming neighborhood. The assignment is to estimate the contribution to market value of a 1,000-square-foot strip of land along the side which will be taken by the local authority for a common landscaping buffer. The appraiser's task is to estimate the subject's value before and after the taking. The difference between the two values is the amount the authority should pay the landowner for the fee simple rights to the strip of land.[1]

Other typical differences between subject's and comparables are designed out of the analysis using the following simplifying assumptions about the comparables:

1. All are in the same neighborhood with no locational differences.

2. None has a better view or traffic noise or other externality that makes it more or less desirable.

3. All are similar to the subject in terms of being level and fully usable and all are similar in terms of zoning, shape, road access, utility access, etc.

4. All sold recently in a level market with adequate exposure. All involved arms-length negotiations with no financing concessions, etc.

Additionally, the strip of land in question is assumed to have no unique features and the proposed landscape buffer does not provide any additional value/desirability. As a result, no severance damages or special benefits or other complications need to be considered.

The following four land sales are provided:

Case Study Data Setå

	Size	Price/SF	Price
L-1	8,000	$11.00	$88,000
L-2	9,000	$10.50	$94,500
L-3	10,000	$10.00	$100,000
L-4	11,000	$9.50	$104,500

Differences in site size impact price in the expected way — that is, larger sites sell for less per square foot and smaller sites for more. The typical method of addressing this is to adjust for these differences, as follows:

	Size	Price/SF	Size Adjust	Adjusted Price
L-1	8,000	$11.00	-$1.00	$10.00
L-2	9,000	$10.50	- $0.50	$10.00
L-3	10,000	$10.00	$0.00	$10.00
L-4	11,000	$9.50	$0.50	$10.00

These sales indicate a value of $10.00/square foot and thus a total site value of **$100,000** (10,000 square foot x $10.00/square foot).

The commonly used approach for estimating the contributing value of the strip is to simply multiply the size of the strip by the same $10/square foot unit price. Using that approach, the contributing value of the strip of land is estimated at **$10,000** ($10.00/square foot x 1,000 square foot).

Unfortunately, the $10,000 estimate is wrong. As will be shown, the correct figure is $5,500. The appraiser made a math error based on their conflation of the following two concepts:

- **Average Unit Price**
- **Marginal Contribution**

These are not the same thing. Assuming they are has resulted in an estimate that is highly flawed.

The Correct Math

The value of the property before it taking is the price per square foot (PSF) x the initial area. As decreasing returns are at play here, the smaller after-taking site has a different (slightly higher), price per square foot (PSF'). Thus, the value after the taking is (PSF') x (Initial area – Taken area) and the contribution of the taken area is as follows:

Contribution = PSF x (Initial area) – PSF' x (Initial area – Taken area)
Contribution = $10.00 SF x 1,000 SF - $10.50 x (10,000 SF – 9,000 SF)
Contribution = $5,500

The correct formula accounts for the fact that the taking increases the average cost per square foot of the remaining lot. This contribution can also be expressed as follows:

PSF x Taken area + [PSF – PFS'] x Initial Area

The first term here is exactly the computation used in the common evaluation method, accounting for the decrease in area at the current price per square foot. The second term above however, is a necessary correction stemming from the fact that if the plot becomes smaller, the average price per square foot of the remaining land increases. Note though that this correction term vanishes when there are no increasing/decreasing marginal returns.

An Easy Way to Conceptualize

Imagine somone going into a grocery store to buy cans of their favorite beverage. They have two options:

1. **6-pack for $7.50 or $1.25/can**
2. **12-pack for $12.00 or $1.00/can**

They decide to purchase the 12-pack. On the way out of the store, they are stopped by a city official who informs them that six of their cans are being taken via the condemnation process. Thus, while they will lose the six cans, they will be compensated for their loss in market value.

If the computation is based on the $1.00/can they paid (average price/can), the compensation is $6.00. However, if it is acknowledged that their remaining six pack now has a slightly higher unit price of $1.25/can and thus a market value of $7.50, the current compensation is as follows:

$12.00 - $7.50 = $4.50

Using the average price can figure would overstate the compensation by 25% ([$6.00-$4.50] / $6.00) meaning this person benefited from the action. He was able to effectively purchase a 6-pack for $6.00 rather than the $7.50 it actually costs in the store.

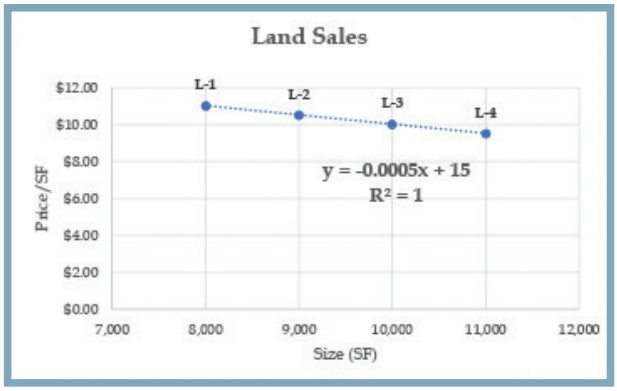
A Simple Method To Avoid The Problem

There is a simple method of approaching such valuations that eliminates the conflation between Average Unit Price and Marginal Contribution. First, adjust comparables for all differences except size. Then, analyze the data graphically. This is demonstrated using the data from above, as follows:

	Size	Price	Price/SF
L-1	8,000	$80,000	$11.00
L-2	9,000	$90,000	$10.50
L-3	10,000	$100,000	$10.00
L-4	11,000	$110,000	$9.50

As shown in Figure 1, these points are graphed in a spreadsheet and appraiser chooses the trendline that best matches the data:[2]

Figure 1



Such analysis can be done by plotting the two relevant sizes on the graph to find the appropriate price/square foot. Another option is to use the formula for each unit value which can be built into a spreadsheet, as follows:

	Size (SF)	Price/SF	Value
Before Taking	10,000	$10.00*	$100,000
After Taking	9,000	$10.50**	$94,500
Difference	1,000		$5,500

*(-0.0005 x 10,000)+15

*(-0.0005 x 9,000)+15

We Are Not That Accurate

A common response to the above by many appraisers is, "We are not that accurate. That it works in the example above tells us little as is uses data manufactured with perfect correlation. The real world is messier and such a nuanced approach relies on an accuracy too refined to reflect the reality of actual markets."

Therefore, the argument continues, "multiplying the Average Price/Unit by the size of the Taking results in a conclusion that is 'close enough.'" It is true that such an approach is "close enough" and in fact mathematically correct in the absence of diminishing returns. This is because Average Price/Unit and Marginal Contribution are constant when decreasing returns are not present.

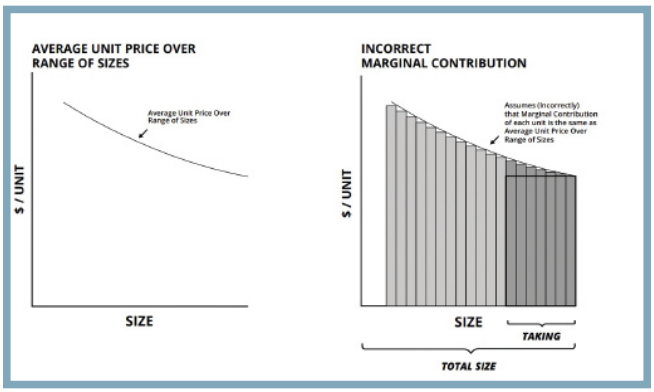
However, when diminishing returns enter the picture, Marginal Contribution becomes non-constant and the simple math no longer works. When this happens "close enough" often becomes "not even close." To demonstrate why, begin with a clear understanding of these two ideas:

Average Unit Price: The total site value divided by the size

Marginal Contribution: What each unit contributes to value over the range of size of a specific property

These are different concepts, but it is easy to confuse them. Such a confusion is demonstrated graphically in the two graphs in Figure 2. That is, the incorrect graph on the right is what is imagined — incorrectly — under the illusion that Average Price/Unit and Marginal Contribution are the same:

Figure 2

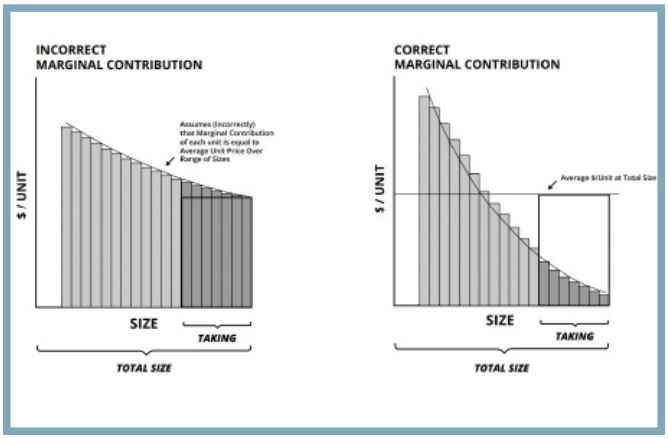


The graph on the left shows a correct Average Price/Unit line. This is a graph showing the relationship between site size and average price/square foot for a property with the qualities of the subject (e.g., location, view, zoning, etc.). Thus, the value of a site with the qualities of the subject can be estimated by plotting the various sizes along this line. The downward slope tells us decreasing returns are at play.

The graph in the right is an incorrect interpretation of what Marginal Contribution would look like if it mirrored Average Unit Price. It does not look like this. If it did, simply multiplying the Average Price/Unit by the Taking could arguably be "close enough" in many situations. As shown in the box around the taking in the graph on the right, only the small incremental difference above the highlighted box would be ignored using that approach.

The problem is that graph on the right is wrong. In Figure 3 below, this same incorrect graph is shown on the left next to a correct graph on the right.[3]

Figure 3



Marginal returns do not mirror Average Price/Unit. They are often quite different.

Since the sum of the areas of the Marginal Return rectangles equal the total site value, only the average of these rectangles is equal to the Average Price/square foot. Thus, if some of the rectangles are above the average line (those to the left of the midpoint in this case), others must be correspondingly lower in order to compensate. Since it is the contribution of the rectangles at the far right that the appraiser is valuating, multiplying the Average Price/Unit by the size of the taking overvalues the contribution.

Closing Thoughts

The main point of this article is that there is a crucial difference between Average Unit Price and Marginal Contribution. Confusing these terms in partial interest valuations, when decreasing returns are present, results in flawed conclusions. The good news is there are easy ways to approach this valuations which resolves these errors.

In Part 2 of this article, which will be printed in the March/April 2025 issue, arguments about appropriate levels of accuracy in appraisal are discussed further. A case is also made for the ubiquity of decreasing returns in most markets. Finally, some arguments for ignoring the ideas set forth this this article are put forth and addressed. ✪

[1] If the authority were taking an easement, the methodology would be similar except that some rights to the land may remain with the underlying landowner.

[2] In Excel, highlight the data points, right click and choose Select Trendline. To include Equation and R-factor, choose those as options.

[3] In real life Marginal Contribution per unit tends to curve as shown in the graph on the right. It is also possible that this line could be straighter or steeper. In lieu of diminishing returns, it would be a straight horizonal line.

Scott B. Arens, MAI, SRA, is the owner of Arens Group, Inc. a real estate appraisal firm in Carlsbad, California. Arens Group provides appraisals with a wide range of property types including significant numbers involving partial takings and easement work (e.g. condemnation, title claims/disputes) as well as expert testimony.

Daniel M. Kane is an associate professor at the University of California San Diego with a joint appointment between CSE and Mathematics. His research interests include number theory, combinatorics and Boolean analysis, though most of his recent work has been in computational statistics.